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Backflow Effects in Nematic Liquid Crystals†

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Abstract—We study the hydrodynamic motion induced by the rotation of the molecular axis in a nematic liquid crystal (backflow). This effect can reduce strongly some effective viscosity coefficients.

When a nematic sample is driven by external torques, the molecules are put into rotation. This rotation may, in turn, induce hydrodynamic motion, called backflow. In spite of many studies on flow alignment, backflow, which is the inverse property, is often neglected.

We discuss these backflow effects, as derived from the Leslie equations,[1] for a few typical geometries:

1) Transient effects in the Freedericks transition of a slab.
2) Motion of walls separating two regions of opposite tilt in a slab under fields above the Freedericks threshold.
3) Sample under an oscillating magnetic field with a free surface.

We show that one has to take backflow into account. Its effects can give important correction on some viscosity coefficients. Our predictions allow the interpretation of the effective frictions measured in experiments (1) and (2). We show also how they could be tested by the motion of dust particles.

1. Transient effects at the Freedericks transition of a slab

Let us start with a case of pure twist (Fig. 3.II). Here the theory predicts that the rotation of molecules does not induce backflows.

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This theory has been tested experimentally by A. Martinet[2] in a different but similar system of magnetic colloids and by L. Léger[3] observing dust particles in the nematic.

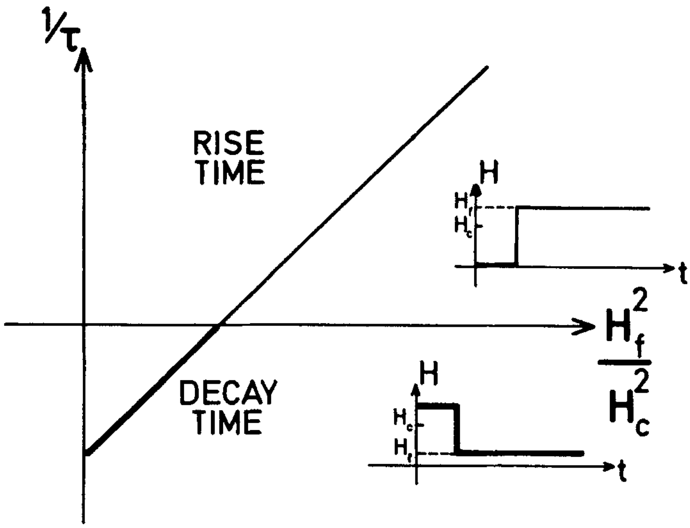


Figure 1. Relaxation rates as a function of H_f^2/H_c^2 in the case of pure twist deformation.

If a stepwise field H (H_f greater than the threshold H_c) is applied (Fig. 1), the distortion starts with an exponential growth[4] with a rate

$$\frac{1}{\tau} = \frac{\chi_a}{\gamma_1}(H^2_{\text{final}} - H_c^2) \tag{1}$$

Here χ_a is the anisotropic part of the diamagnetic susceptibility and γ_1 the twist viscosity coefficient.

On the other hand, if the field H is decreased to a value less than H_c, the distortion decays exponentially. The time constant is still given by the same expression if we describe the decay process by the negative value of $1/\tau$.

In the homeotropic geometry (Fig. 3.III) the coupling between the rotation of the molecules and flow is very strong and backflow is induced. The flow velocity contains two components (Fig. 2), one

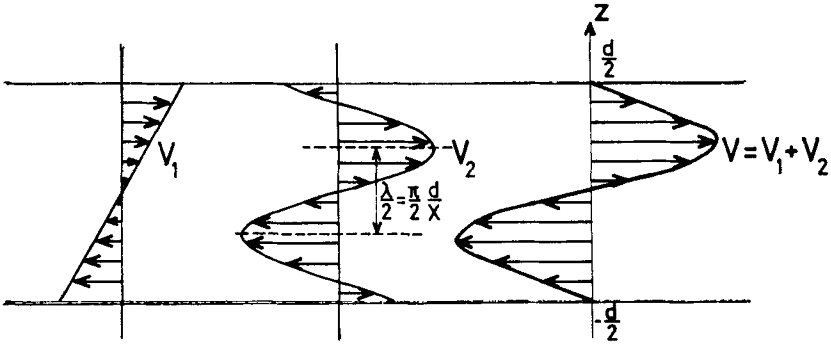


Figure 2. Flow velocity in the homeotropic to planar transition.

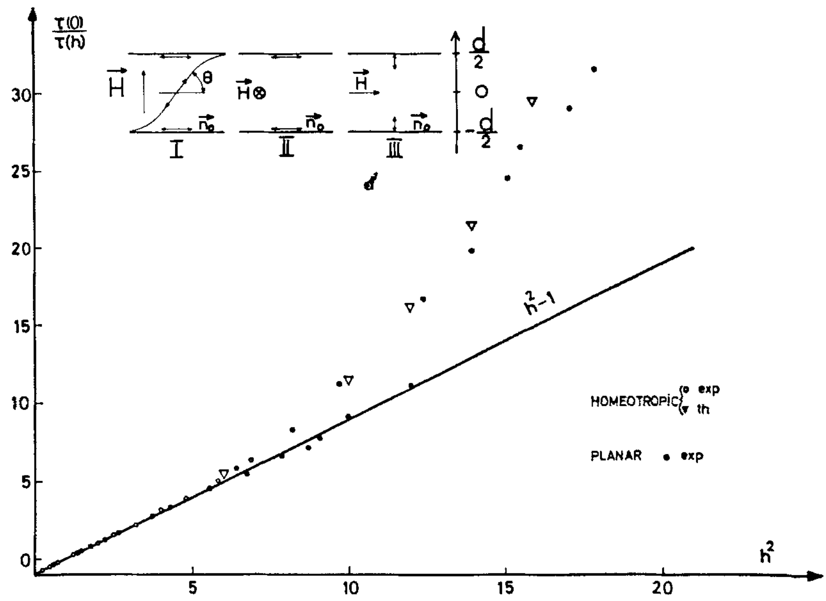


Figure 3. Theoretical and experimental normalized relaxation rates as a function of $h^2 = H_f^2/H_c^2$ for geometries I (planar case), II (twist case), III (homeotropic case). H is the magnetic field; $\mathbf{n}_0$ labels the optical axis of the unperturbed configuration. In geometry III, the deviation from a parabolic variation provides a spectacular demonstration of the backflow effects.

which is linear in z, and one which is sinusoidal. The wavelength λ of this sinusoidal component depends on the field H_{final}.[5] λ is a decreasing function of H_f.

The rise time and decay time are still given by Eq. (1), but now $\gamma_1{}^*$ is a function of H_f[5]. For *p*-methoxybenzylidene-*p*-*n*-butylaniline (M.B.B.A.) $\gamma_1{}^*/\gamma_1$ decreases from 0.85 to 0.25 when H varies from 0 to ∞. In Fig. 3, we show the theoretical and experimental relaxation rates for the homeotropic to planar transition. The experimental points by Guyon and Pieransky are obtained by monitoring the transient distortion by a conoscopic technique. There is a rather good agreement between the two sets. Note that the plots are now curved upwards; this is to be contrasted to the simple law for twist, which may be represented by a straight line. The curvature indicates the importance of backflow.

Let us now mention the planar to homeotropic transition (Fig. 3.I). Here, for fields H near the threshold, the molecules are not efficient in inducing backflow. This is illustrated in Fig. 4.

Two remarks should be made at this point:

(1) for all geometries where backflow is present, the apparent friction constant $\gamma_1{}^*$ is smaller than γ_1; backflow helps to relax the constraint.

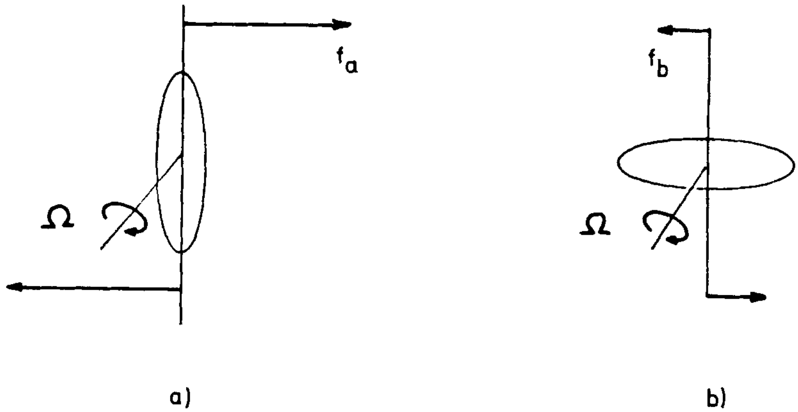


Figure 4. In the presence of the rotation of the molecules with an angular velocity the torque acting on an elementary volume of the fluid is

$$f_a = \frac{\gamma_1 - \gamma_2}{2}\Omega, f_b = \frac{\gamma_1 + \gamma_2}{2}\Omega. \text{ Usually } |f_a| \gg |f_b|.$$

(2) the importance of backflow effects is very sensitive to the exact boundary conditions at the sample surface. Consider, for instance, this slab with one free surface and homeotropic alignment. Under an horizontal field H, we obtain for the velocity profile the form described in Fig. 5. There is now a non zero average velocity in the sample. Here backflow is particularly large and the expected reduction of $\gamma_1{}^*$ is considerable ($\gamma_1{}^* \sim \gamma_1/4$ for M.B.B.A.). Unfortunately, for this case, we have no quantitative measurements of $\gamma_1{}^*$.

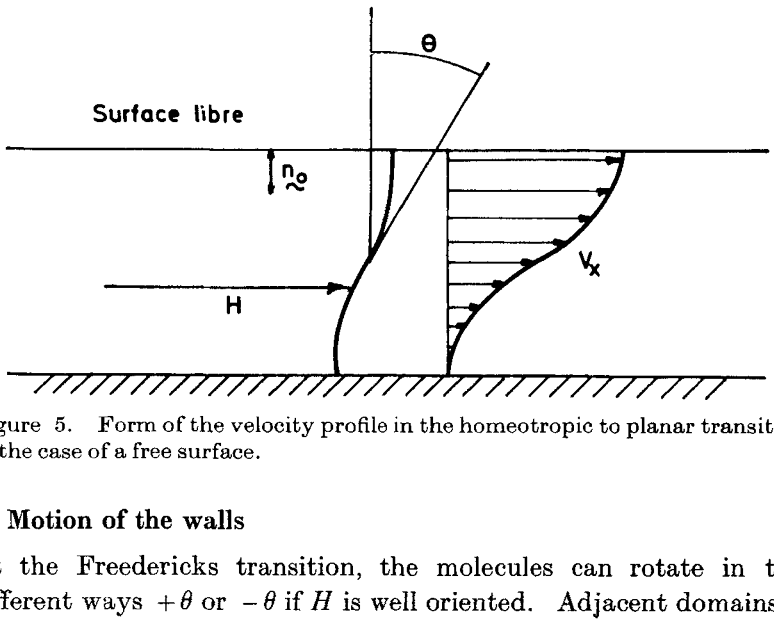


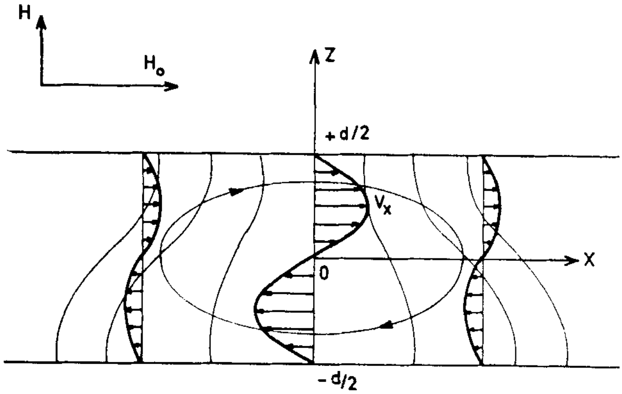
Figure 5. Form of the velocity profile in the homeotropic to planar transition in the case of a free surface.

2. Motion of the walls

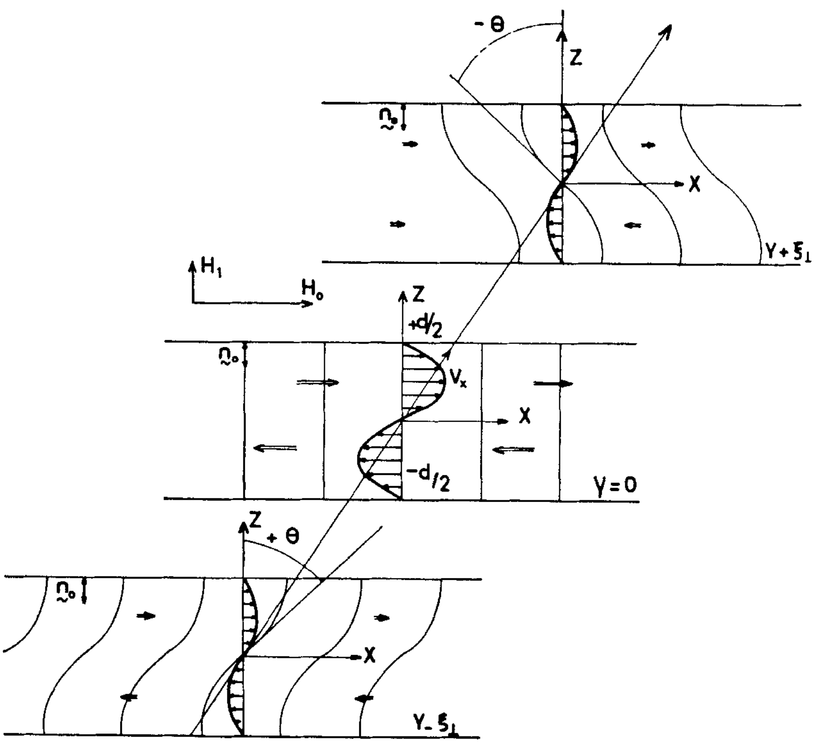
At the Freedericks transition, the molecules can rotate in two different ways $+\theta$ or $-\theta$ if H is well oriented. Adjacent domains of opposite tilt are separated by a wall,[6] that is to say a region in which the molecular axis rotates gradually from $-\theta$ to $+\theta$.

Now, if we tilt the magnetic field H slightly, the walls move to enlarge the favoured domains. You see in Fig. 6 the backflow induced by the motion of a wall parallel and perpendicular to H. In that case, one can see dust particles move along with the wall.

The backflow reorients the molecules and the mobility of the wall is increased. By comparing the mobility of the walls in the homeotropic ($\gamma_1{}^*$) and planar (γ_1) cases, it is possible to check the ratio $\gamma_1{}^*/\gamma_1$. The experimental result[7] $\gamma_1{}^*/\gamma_1 = 0.85$, close to H_c. This value is in good agreement with theoretical predictions.



(a)



(b)

Figure 6. Backflow induced by the motion of a wall perpendicular to H (case a) and parallel to H (case b).

3. Sample under Oscillating Magnetic Field with a Free Surface

A magnetic liquid crystal in a strong vertical magnetic field H_0 is submitted to a small horizontal oscillating field $H_1 \cos \omega t$. We consider a sample with a free surface. With this boundary condition the backflow effects are a maximum.

In the case of a thin slab of thickness d, the stress is conserved (and equal to zero in the case of a free surface) if $d \ll (\eta/\omega\rho)^{1/2}$. We obtain for the velocity profile the form described in Fig. 7. The velocity is linear in z.

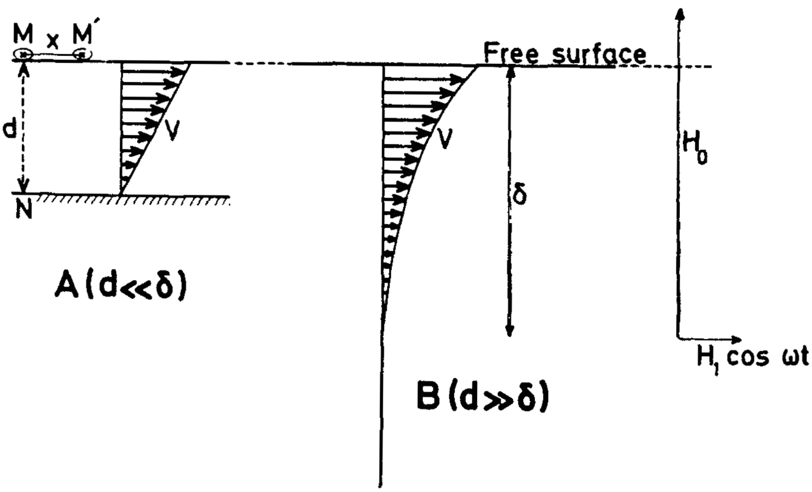


Figure 7. Backflow induced by an oscillating magnetic field: A: in a slab B: in a bulk sample.

The motion of the fluid can be tested by observing the displacement x of a solid particle floating on the surface. The displacement is defined as

$$X = \frac{\alpha_2}{\eta_2}\frac{H_1}{H_0} d \cos(\omega t + \phi) \cos\phi$$

with $\operatorname{tg} \phi = \omega\tau_0{}^*$, $\tau_0{}^* = \gamma_1{}^*/\chi_a H_0{}^2$.

($\gamma_1{}^* \sim \gamma_1/4$ for M.B.B.A.)

In a bulk sample, the flow penetrates only a distance $\delta \sim (\eta/\omega\rho)^{1/2}$

where η is a viscosity. The displacement of a particle is still given by an expression similar to (2), with d replaced by δ.

Conclusion

It may seem surprising that, in spite of many studies on flow alignment, backflow, which is the inverse property, is often neglected. It has been shown here that in many experiments one has to take backflow into account and that its effects can give important correction on some viscosity coefficients.

On the other hand, backflow leads to a very simple experimental test of Leslie's theory. From the study of backflow effects, we can also derive some viscosity coefficients.

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